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Experimental Investigations of Electromagnetic Instabilities of Free Surfaces in a Liquid Metal Drop

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Abstract

High-frequency electromagnetic fields are able to shape the surface of molten metal when applied to the surface. In some cases, this effect can be used to considerably reduce the convective heat losses within melts. To realize this technique, it is necessary to analyze the surface shape and its stability. A simple system of plate-type induction coil and molten metal is used in the experiments. The liquid metal is arranged in a shell–like water-cooled container made of glass. The free surface contour is observed by both a CCD camera and a high-speed camera system while varying the inductor current and the frequency. The data is compared with predictions of both an analytical model and numerical simulations. Details of the numerical investigation are given in another MEP-paper "Numerical Simulation of a Liquid Metal Drop under the Influence of Lorentz Forces" (U. Lüdtke). When the inductor current exceeds a certain critical value, the drop starts to pulsate. Bumps and dents are traveling along the perimeter. The dynamics of the surface fluctuations are recorded. A weak formulation is used to derive an equation for the evolution of surface perturbations. Fluctuation phenomena are explained. The aim of the present study is to resolve how electromagnetic fields cause surface fluctuations of molten metals and how the surface shape and the stability are affected.

Introduction

The deformation of a liquid metal drop in a high-frequency electromagnetic field was studied by Kocourek et al. [1]. The authors show that electromagnetic forces can be effectively used to hold the drop in shape. The Lorentz forces are generated by an inductor that is arranged as shown in Fig. 1. The inductor is fed by a high-frequency electrical current. Kocourek et al. [1] observe that upon increasing the inductor current beyond a certain value, the nearly circular shape of the drop becomes unstable. Fautrelle et al. [2] show experimental results in the case that a low-frequency electromagnetic field acts on the drop. They observe that under the influence of the field, an initially circular drop may fall apart into several smaller drops via various instabilities. The final diameter of the drops corresponds to the capillary length scale. Furthermore, they observe that a drop may undergo a deformation into a torus-like shape before exploding in a "big bang". Fautrelle and Sneyd [3], [4] present a stability analysis of a flat liquid metal surface exposed to an alternating magnetic field. Using a classical perturbation expansion they show that the instability can be described by a Mathieu Hill equation. Mohring and Karcher [5] have experimentally and analytically investigated the stability of a free liquid metal interface subject to an applied magnetic pressure. In the experiments they observe that an electromagnetic pinch sets in when the feeding current exceeds a certain critical value. A simple linear stability analysis based on Hele Shaw and skin-depth approximation predicts that the critical current I_C scales as $I_C \propto w^{-1/4}$ and $I_C \propto L$, where ω denotes the frequency and L the gap between inductor and surface.

The main purpose of this paper is to study the instability of the drop by varying frequency. The paper is organized as follows: In Sec. 1 we describe the experimental set-up. Some experimental results are shown in Sec. 2. In Sec. 3 we present a simple linear stability model and in Sec. 4 we compare the experimental findings with theoretical predictions.

1. Experimental set-up

A principle sketch of the experimental set-up is shown in Fig. 1. It consists of a cylindrical container with an inner diameter of D = 60 mm within which a curved glass plate is arranged. The radius of curvature of the plate is r_c = 80 mm. In the experiments, we place a certain volume of liquid metal on the glass plate. As a test liquid we use GaInSn in such a composition that its melting point is as low as -19°C (Galinstan). The liquid metal shows the following properties at 20°C: electrical conductivity $\kappa = 3.46\ 10^6$ S/m, density $\rho = 6440$ kg/m^3, surface tension $\sigma = 0.718$ N/m, and a static contact angle of $\Theta = 125°$. However, due to the curvature of the plate, the actual contact angle between drop and plate is less. To provide quasi-isothermal conditions during the experiments, the bottom of the glass plate is water-cooled. The cooling allows to remove Joule heat losses of up to 50 W out of the drop. During the experiments we cover the drop with hydrochloric acid to avoid oxidation of the free surface. The electromagnetic field is generated by an inductor arranged at the same height as the drop. The inductor is made of copper and consists of 10 windings arranged in two layers with an inner radius of r = 48 mm. The inductor is fed by an alternating electrical current. In the experiments we increase the feeding current up to 350 A and vary its frequency within the range 500 Hz < f< 50 kHz.

During the experiments we observe the deformation of the drop from above via both a CCD and a high-speed camera system at a rate of 307.7 fps. The electrical current is measured by two different gages. However, for currents I > 100 A and frequencies f > 20 kHz the readings of the gages differ by about 10%.

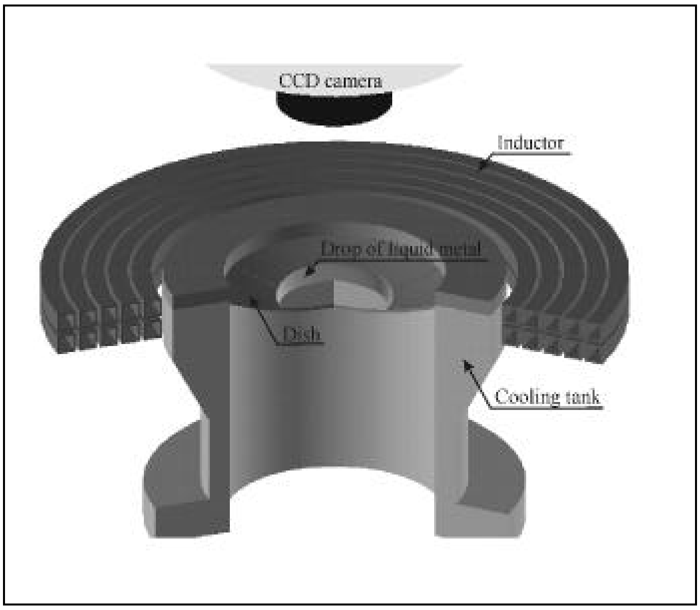

Fig. 1. Sketch of experimental set-up.

2. Experimental results

In the following we show results of an experimental run where the drop volume is fixed at V = 3.4 cm^3 resulting in an initial drop radius of r = 14.52 mm when the magnetic field is absent.. As an example, Figs. 2 and 3 show how the drop is deformed when a critical inductor current of frequency f = 8 kHz and f = 31.2 kHz is applied. We observe that the drop contour show an azimuthal mode number n = 2 for frequencie up to 10 kHz and mode number n = 3 for higher frequencies. The data for critical values of the applied current at frequency of 2.1 kHz < f < 43.2 kHz is shown in Fig. 5. The unstable drop rotates and pulsates along the perimeter. Table 1 summarizes the experimental observations.

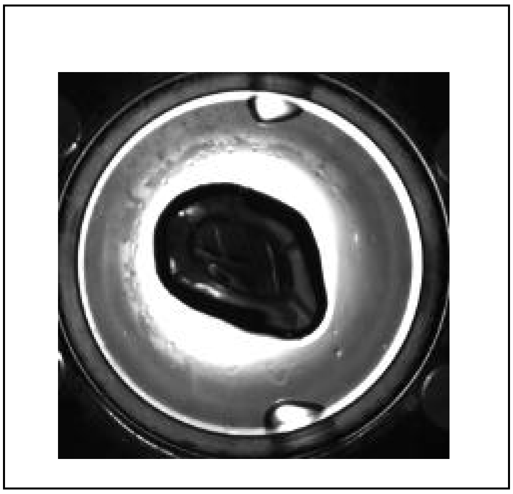

Fig. 2. Instability at f=8kHz and I=Ic=187A

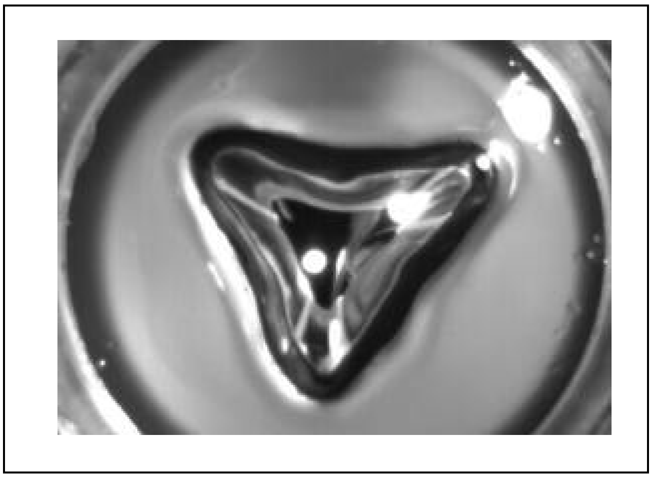

Fig. 3. Instability at f=31.2kHz and I=Ic+40A=182A

Tab. 1. Instability data

Applied frequency	f=8kHz	f=22.3kHz		f=31.2kHz		f=43.2kHz
Applied current	I=Ic=187A	I=Ic+20A =171A	I=Ic+40A =191A	I=Ic+20A =162A	I=Ic+20A =182A	I=Ic+20A= 155A
Rotation frequency in Hz	-	4.44	5.3	4.53	4.23	**4.5**
Mode	2	3	3	3	3	3
λ in mm	39,1	25.1	25.9	26.1	25.2	26.5
r_{max} in mm		14.1	16.3	14.3	15.9	1.37
r_{min} in mm		11.1	8.4	10.6	8.2	10.7

3. Theoretical analysis

3.1. Governing equations

To support the experimental observations we apply the same linear stability analysis as in [5]. The geometry of the problem is shown in Fig. 4. Plane liquid metal interface in a Hele-Shaw cell is subjected to a high-frequency magnetic field. The magnetic field is generated by an inductor located at distance z = L above the unperturbed surface. The inductor is fed by an alternating current $I\cos\omega t$ pointing in y-direction. Since the skin depth $d_S = \sqrt{2/(swm_0)}$ is small as compared with the liquid metal height H, the magnetic field B can be calculated using the mirror image method.

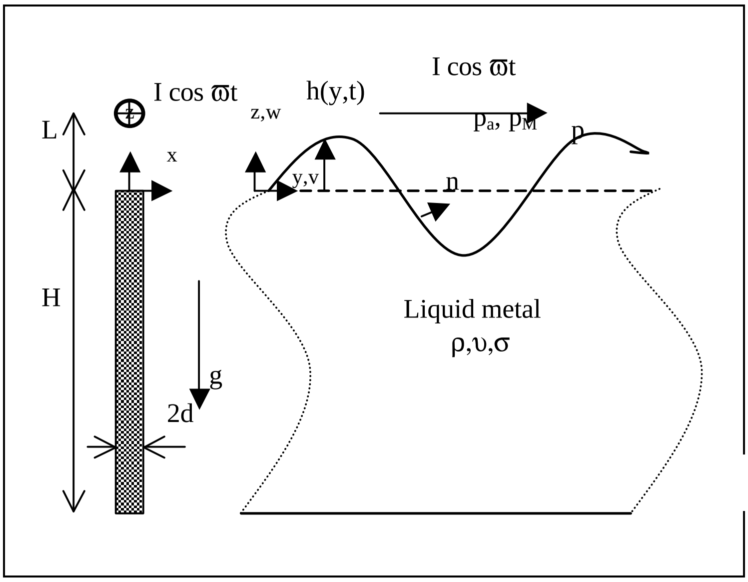

Fig. 4.: Sketch of the problem used in the analytical modeling

Furthermore, we apply the usual Hele-Shaw approximation (Schlichting [6]) that the velocity field **v** = (u,v,w) within the liquid metal can be represented by

$$u = 0,\ v = v(y,z,t)\cdot\left(1-\frac{x^2}{d^2}\right),\ w = w(y,z,t)\cdot\left(1-\frac{x^2}{d^2}\right),$$

and that porous friction $\propto \upsilon / d^2$ is dominant. Within these assumptions, the coupled hydrodynamic and electrodynamic equations read as follows:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0,\ \frac{\partial v}{\partial t}=-\frac{1}{r}\frac{\partial p}{\partial y}-u\,\frac{2}{d^2}v,\ \frac{\partial w}{\partial t}=-\frac{1}{r}\frac{\partial p}{\partial z}-u\,\frac{2}{d^2}w-g,$$

$$\oint_c \frac{B}{m}\,dl = I.$$

Additionally, at the interface at z = h the following pressure condition holds:

$$p_M + p_a - p = S\frac{\partial^2 h}{\partial y^2}\left(1+\left(\partial h/\partial y\right)^2\right)^{-3/2}.$$

Here, S is surface tension and p_a, p_M represent the ambient pressure and the magnetic pressure. Once the magnetic field B is known, p_M can be calculated using the relations

$$p_M = \frac{\left.\left\langle \mathbf{B^2}\right\rangle\right|_{z=h}}{2m_0}$$, where $\langle\ \rangle$ denotes time averaging.

3.2. Linear stability analysis

We now introduce the perturbation expansion $\Phi = \overline{\Phi} + e\Phi_1,\ \Phi = \in (v, w, p, A, h, p_M)$, into the governing equations and respective boundary conditions and collect terms of equal power in ε. For the leading order we obtain the basic state given by

$$\bar{v} = \bar{w} = \bar{h} = 0;\ \bar{B} = \frac{m_0 I}{2p}\left[\frac{(L-z)}{(L-z)^2 + x^2} + \frac{(L+z)}{(L+z)^2 + x^2}\right];\ \overline{p_M}(z=0) = \frac{m_0 I^2}{2p^2 L^2}.$$

In the next order we find that the problem is described by Laplace equation according to

$$\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} = 0;\ w_1(z=-H),\ w_1(z=0) = \frac{\partial h_1}{\partial t}.$$

We solve the above equations by introducing normal modes of the form $w_1 \propto \cos ky$. Inserting the solution into the equation

$$\frac{\partial w_1}{\partial t} + \frac{2u}{d^2} w_1 = -\frac{1}{r}\frac{\partial p_1}{\partial z},\ \text{where}\ p_1 = Sk^2 h_1 + rgh_1 - \frac{\bar{B}B_1}{m_0};\ B_1 = \frac{\partial \bar{B}}{\partial z} h_1,$$

and follow-up integration we obtain

$$\frac{r}{k}\frac{1+\exp(-2kH)}{1-\exp(-2kH)}\left\{\frac{\partial^2}{\partial t^2} + \frac{2u}{d^2}\frac{\partial}{\partial t}\right\}h_1 + rg\,h_1 = -Sk^2 h_1 + \frac{m_0 I^2}{2p^2 d_S L^2} h_1.$$

It is obvious that the most dangerous modes are characterized by k = 0 as these modes are not stabilized by surface tension. In this case, the criterion for the onset of neutral instability $(\partial/\partial t = 0)$ reads as

$$I_c^2 = \frac{2p^2}{m_0} L^2 rgd_S.$$

3. Comparison

Fig. 5 compares the predictions of the analytical model for the critical inductor current with the experimental findings. We observe that there is a good agreement for higher values of the applied current frequency. However, for lower values of f (f < 5 kHz) the measured curve does not show the typical dependence $I_C \propto w^{-1/4}$. This deviation can be explained as a

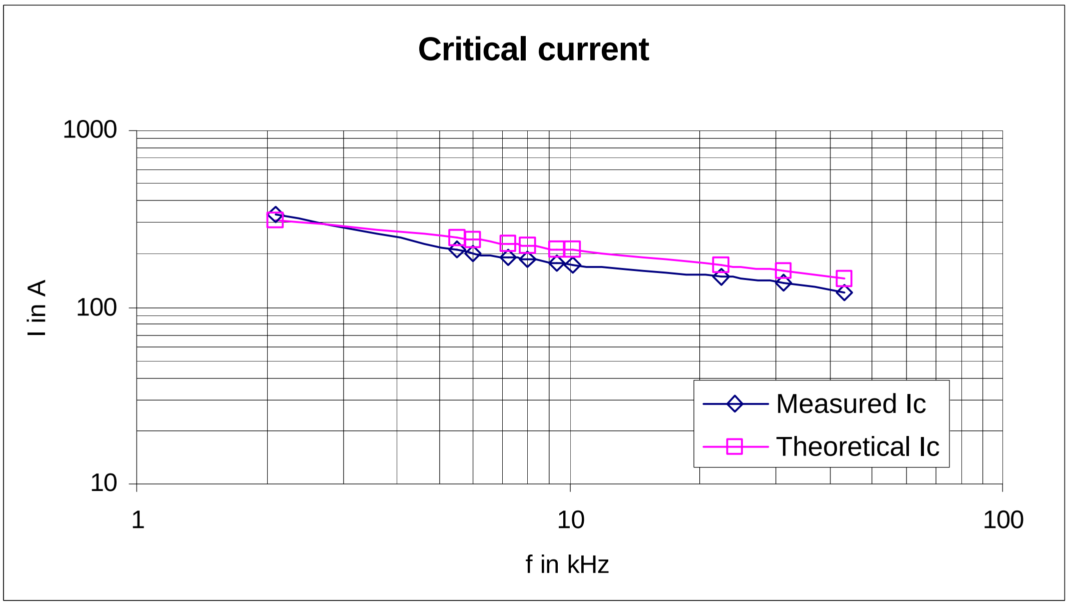

Fig. 5. Comparison of measured and calculated Ic

violation against the model assumption, i.e. that the skin depth d_S is very small compared to the liquid metal height H.

Fautrelle et al. [4] show that the azimuthal mode number n increases when the frequency is increased. We observe this tendency as well with the transition from n = 2 to n = 3 at frequency of about 10 kHz. We might expect another transition to higher mode numbers at frequencies beyond 50 kHz.

Conclusions

We have experimentally and analytically investigated the stability of a liquid metal drop in a high-frequency alternating magnetic field. We find reasonable agreement between the predictions of a simple stability model and the experimental observations. Our findings demonstrate that the onset of the instability can be explained as follows: Instability sets in when the induced magnetic pressure $p_M \propto \frac{B^2}{2m_0}$ at the interface equals the hydrostatic pressure evaluated with the skin depth, i.e. $p \propto rgd_S$.

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